Sparse Representation of Electrodermal Activity With Knowledge-Driven Dictionaries

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Abstract—Biometric sensors and portable devices are being increasingly embedded into our everyday life, creating the need for robust physiological models that efficiently represent, analyze, and interpret the acquired signals. We propose a knowledge-driven method to represent electrodermal activity (EDA), a psychophysiological signal linked to stress, affect, and cognitive processing. We build EDA-specific dictionaries that accurately model both the slow varying tonic part and the signal fluctuations, called skin conductance responses (SCR), and use greedy sparse representation techniques to decompose the signal into a small number of atoms from the dictionary. Quantitative evaluation of our method considers signal reconstruction, compression rate, and information retrieval measures, that capture the ability of the model to incorporate the main signal characteristics, such as SCR occurrences. Compared to previous studies fitting a predetermined structure to the signal, results indicate that our approach provides benefits across all aforementioned criteria. This paper demonstrates the ability of appropriate dictionaries along with sparse decomposition methods to reliably represent EDA signals and provides a foundation for automatic measurement of SCR characteristics and the extraction of meaningful EDA features.

Index Terms—Dictionary design, electrodermal activity, (orthogonal) matching pursuit, skin conductance response, sparse representation.

I. INTRODUCTION

RECENT technological achievements are enabling the increasing use of wearable devices that allows the sensing of physiological signals for health [1], [2], medical [3], [4] and other [5], [6] purposes. These applications stem from the need to monitor individuals over long periods of time overcoming the limits imposed by traditional nonambulatory technology and providing new insights into diagnostic and therapeutic means [3]. They typically rely on small sensors for capturing data, portable devices for temporal storage, and the use of wireless networks for (periodic) data transfer to a database [7]. The large volume of recordings from these sensors, their use in everyday life and beyond specialized places like clinics, the potential variability in the data quality, and the limited resources of appropriately trained people for the corresponding signal analysis underscore the need of automatic ways to process them. A central goal is to derive meaningful quantitative constructs from the measured signal data. Robust representations of physiological signals should consider their characteristic structure, efficiently encode the underlying information, and provide reliable interpretation.

Electrodermal activity (EDA) is one of the most widely used psychophysiological signals. It is related to the sympathetic nerve activity through the changes in the levels of sweat in the ducts [8], [9], that result in the observed skin conductance response (SCR) fluctuations. EDA has been used in a variety of laboratory settings examining attention, memory, decision making, emotion, as well as a predictor of normal and abnormal behavior and other psychological constructs [8], [9].

Recently the focus has shifted to collecting EDA signals outside the laboratory “in the wild” [10], [11] with wearable devices that can measure continuously, privately logging and/or wirelessly streaming data [12]–[14]. This unobtrusive long-term recording of EDA results in large amounts of data that require deriving appropriate signal representation and interpretation with two separate goals. EDA models should first target efficient signal compression yielding reduced memory allocation and fast network transfer, accompanied with low reconstruction error. Second, since the availability (and analysis ability) of human experts is not always guaranteed, in the light of human–machine applications, automated interpretation of the acquired physiological patterns is essential. The reliable detection of SCRs, linked to various indexes of psychophysiological interest [8], can serve as a step toward this goal. Eventually, inference of internal states based on EDA representations could afford us more insights by comparing the parameters of the corresponding models across various conditions of emotion, arousal, and attention [15], [16].

Previous research has introduced parameterized [17] and generative causal [15], [18], [19] models that consider the characteristic structure of the signal or mimic the physiological processes of the sweat ducts. These methods tend to explain the cause and not the observed signal. They are mostly evaluated through empirical findings about hidden variables for inferring internal states [20], although several efforts have taken into account signal reconstruction criteria [15], [16].
Sparse representation techniques model a signal as a linear combination of a small number of atoms chosen from an overcomplete dictionary aiming to reveal certain structures of a signal and represent them in a compact way [21]. Since psychophysiological signals, such as EDA, show typical patterns over time, their sparse decomposition can yield accurate representations of scientific and translational value and contribute to scalable implementations (e.g., on mobile devices). Noting that the desired information contained in EDA signals is low dimensional, we introduce the use of sparsity to robustly represent them. The innovation of our approach lies in that we model the EDA signal directly by taking into account its variability through the use of overcomplete knowledge-driven dictionaries and evaluate our method explicitly through both signal reconstruction and information retrieval measures.

We propose EDA-specific dictionaries that take into account the characteristic signal structure in time. Dictionaries include tonic and phasic atoms capturing the signal levels and SCR fluctuations, respectively. We represent the characteristic SCR shape with previously used sigmoid-exponential [17] and Bateman functions [15], as well as a chi-square function, introduced in this study, and decompose EDA into a small number of tonic and phasic atoms with matching pursuit [22] and orthogonal matching pursuit [23]. Through postprocessing of the selected phasic atoms, we automatically detect the SCRs occurring in the signal. We evaluate the usefulness of our approach through goodness of fit criteria and SCR detection measures with respect to human annotated groundtruth. Due to potential portable device applications for signal storage and transmission, we further analyze the compression rate of our algorithm. Our results indicate that the proposed approach provides benefits compared to the least-squares fit method that estimates predetermined model parameters [17], with respect to reconstruction, discrimination, and efficiency metrics.

In the following, we briefly survey previous EDA models (see Section II). In Section III, we present our sparse EDA representation approach with knowledge-driven dictionaries. We next describe the data for this study (see Section IV) and the corresponding results (see Section V). Finally, we discuss our results and conclusions in Sections VI and VII, respectively.

II. BACKGROUND AND RELATED WORK

The EDA signal is decomposed into tonic and phasic components. The slow moving tonic part, called skin conductance level (SCL), depicts the general trend, whereas the fast fluctuations superimposed onto the tonic signal are the skin conductance responses (SCR) (see Fig. 1) [8]. The shape of SCRs is characterized by a steep increase in the signal and a slow recovery [24]. Amplitude is the most commonly reported SCR measure, quantified as the amount of increase in conductance from the onset of the response to its peak (see Fig. 1) with typical values ranging between 0.1 and 1.0 μS [8]. SCRs are caused by the burst of the sympathetic sudomotor nerves controlling the sweat glands linked to emotion, arousal, and attention [8]. Previous work has modeled explicitly the shape of the resulting signal with appropriate functions [17], [25] or implicitly the causal relation between the underlying activity of sudomotor nerves and the observable SCRs [15], [18], [19]. A review on EDA models can be found in [19].

Several studies have mathematically expressed the SCR shape. Lim et al. [17] developed a parameterized sigmoid-exponential model of EDA fitted into signal segments. Results were found to be correlated with previously established automatic scoring methods [24]. Storm et al. [25] used a quadratic polynomial fit to sequential groups of datapoints to detect SCRs, whose total number was compared to manual counting.

In the context of causal EDA modeling, Alexander et al. [18] represented the SCR shape as a biexponential function and used deterministic inverse filtering to estimate the driver of nerve bursts. Evaluation of the method was performed by visual inspection and by finding significant correlations of the resulting SCR measures with variables of gender and age. Benedek et al. [15], [26] assumed EDA to be the convolution of a driver function, reflecting the activity of sweat glands, with an impulse response depicting the states of neuron activity. This method was evaluated through the reconstruction error. It was also compared to standard peak detection for a set of noise burst events and was found to give results more likely to confound with these environmental conditions. Finally, Bach et al. [16], [19], [27], [28] have described a dynamic causal model (DCM) using Bayesian inversion to infer the underlying activity of sweat glands. Each sudomotor activity burst is modeled as a Gaussian function, which serves as an input to a double convolution operation yielding the EDA signal. The correlation of the estimated bursts with the number of SCRs from semisupervised analysis was reported. This model was found to be a good predictor of anxiety in public speaking.

EDA decomposition was cast as a convex optimization problem in [29]. The minimization of a quadratic cost function was used to estimate the tonic and phasic signal components, represented with a cubic spline and a biexponential Bateman function, respectively. The EDA features extracted from the model parameters yielded statistically significant differences between neutral and high-arousal stimuli.

Despite their encouraging results, some of these research efforts [17], [25] tend to impose restrictions on the signal structure. Also, studies modeling EDA through its relationship with sympathetic arousal [15], [18], [19] assume a linear-time invariant system, which is not always justified by empirical evidence [20]. While several studies take into account signal reconstruction measures [15], [16], evaluation is mostly performed by visual inspection or implicitly through expected empirical
assumptions correlating the systems’ SCR measures with physical, mental, and behavioral states. The novelty of this study lies in the fact that it directly models the EDA signal with sparsity constraints and takes into account the SCR shape variability. We evaluate our approach through both signal reconstruction criteria and measures comparing automatically detected SCRs to human-annotated SCRs.

### III. PROPOSED APPROACH

Many psychophysiological signals carry distinctive structures in time making the use of sparse decomposition techniques appealing. Their small number of nonzero components contain important information about the signal characteristics, which can potentially be related to various medical conditions and psychological constructs. For this reason, dictionaries have to be carefully designed so that they capture the signal variability and their underlying information. We propose the use of parameterized EDA-specific dictionaries that are able to represent the tonic and phasic parts of the signal (see Section III-A). The benefit of introducing these predetermined atoms is that we know their properties in advance, such as amplitude and rise and recovery time, which are of interest in a variety of psychophysiological studies [30]. We further decompose EDA into a small number of atoms (see Section III-B) which are used to retrieve important information about the signal characteristics, which can be used in wider SCRs and vice-versa (see Fig. 2). Since there is no complete) dictionaries in sparse representation techniques, this function benefits computational cost and memory allocation, an important factor for mobile device applications. In order to ensure large variety in the dictionary atoms, we considered two additional parameters common for all phasic atom types. The time scale $s$ is responsible for the compression and dilation of the atom in time with lower values resulting in wider SCRs and vice-versa (see Fig. 2). Since there is no a priori information about the location of SCRs within an analysis frame, the dictionary contains shifted versions of all atoms over a range of time offsets $t_0$ spanning one signal frame.

We will refer to the three types of sigmoid-exponential, Bateman, and chi-square SCR atoms as $g_{\gamma(1)} \in \mathbb{R}^L$, $g_{\gamma(2)} \in \mathbb{R}^L$, and $g_{\gamma(3)} = \chi^2(\sigma t - \lambda, k)u(t - t_0)$, respectively; the higher they are, the less time it takes for the SCRs to transition to and from a peak. Finally, the probability density function of the chi-squared distribution, referred here as chi-square function, is introduced by the authors of this paper as an alternative way to represent the SCR shape. It involves only one parameter $\sigma$, with large values resulting in wider SCRs. Although no apparent physiological explanation could be found for the chi-square, each dictionary atom is encoded with one fewer parameter. Taking into account the large size of (over-complete) dictionaries in sparse representation techniques, this function benefits computational cost and memory allocation, an important factor for mobile device applications.

### A. Dictionary Design

Since our approach involves a knowledge-based representation of signals, the choice of dictionary atoms is critical. Specifically for EDA, we introduce two kinds of atoms that model the tonic and phasic components of the signal (see Table I).

The first group takes into account the tonic part, i.e., the slow varying signal level. It is quantified with straight lines having an offset $\Delta_0$ and a slope $\Delta$. Tonic atoms will be referred as $g_{\Delta} \in \mathbb{R}^L$, where $\beta = (\Delta_0, \Delta)$ is an element of the set $\mathbb{B} = \mathbb{R}^2$. Negative values of $\Delta$ indicate decreasing lines, while positive values represent lines with increasing slope.

The second group of atoms captures the phasic component of EDA, meaning the rapid fluctuations superimposed on the signal, also known as SCRs. We use three types of functions that can potentially represent the characteristic steep rise and slow decay of SCRs: the sigmoid-exponential, the Bateman, and the chi-square functions. The sigmoid-exponential was introduced in [17] for representing SCRs with high values of $T_{\text{rise}}$ and $T_{\text{decay}}$ resulting in long rise and decay time. The Bateman function [15], inspired by the laws of diffusion at the skin pores, represents the shape of an impulse response, which when convolved with a driver function results in the EDA signal. Parameters $a$ and $b$ control the steepness of recovery and onset, respectively; the higher they are, the less time it takes for the SCR to transition to and from a peak. Finally, the probability density function of the chi-squared distribution, referred here as chi-square function, is introduced by the authors of this paper as an alternative way to represent the SCR shape. It involves only one parameter $\sigma$, with large values resulting in wider SCRs.

### Table I

<table>
<thead>
<tr>
<th>Atom</th>
<th>EDA part</th>
<th>Equation</th>
<th>Parameter Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Straight Line</td>
<td>Tonic</td>
<td>$g_\Delta(t) = \Delta_0 + \Delta \cdot t$</td>
<td>$\Delta_0 \in {-20, -10, 1}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$\Delta \in {-0.01, -0.009, \ldots, -0.001, 0, 0.01, 0.02, \ldots, 0.1}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$T_{\text{rise}} \in {0.5, 0.8, \ldots, 2.9}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$T_{\text{decay}} \in {0.3, 0.6, \ldots, 3}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$s \in {0.06, 0.07, \ldots, 0.14}$</td>
</tr>
<tr>
<td>Sigmoid-exponential</td>
<td>Phasic</td>
<td>$g_{\gamma(2)}(t) = e^{-x(t-t_0)/T_{\text{decay}}} - e^{-a(t-t_0)} u(t-t_0)$</td>
<td>$a \in {0.2, 0.3, \ldots, 2}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$b \in {0.4, 0.6, \ldots, 2}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$t_0 \in {0, 0.1, 0.2, \ldots, 310}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$k \in {2.70, 2.73, \ldots, 5.37}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$u(t) = 1, t \geq 0$, and $u(t) = 0$ otherwise, $\chi^2(t, k)$: the value of $k$-degrees chi-square distribution at point $t$</td>
<td></td>
</tr>
</tbody>
</table>
 Sparse Representation of a Signal $f \in \mathbb{R}^L$ can be described by the following equation providing a nonconvex problem [21]:

$$\min_{c} \|c\|_0 \quad \text{subject to} \quad f = Dc$$  \hspace{1cm} (1)

where $D \in \mathbb{R}^{L \times Q}$ is an overcomplete dictionary matrix with $Q$ prototype signal-atoms and $c \in \mathbb{R}^Q$ are the atom coefficients with $N \ll Q$ nonzero elements. One way to approach this NP-hard problem is to use greedy strategies, that abandon exhaustive search in favor of locally optimal updates. The most well known are MP [22] and OMP [23], [31] reaching suboptimal solutions. Another way of solving Eq. (1) is through relaxation of the discontinuous $l^0$-norm leading to basis pursuit [32] and focal underdetermined system solver [33], [34]. Despite the fact that these relaxation approaches can obtain global solutions, they are far more complicated and less efficient in terms of running time [21], rendering MP algorithms compelling because of their simplicity and low computational cost. Theoretical guarantees of correctness [35] along with empirical experiments [36]–[38], led us to the use of MP and OMP in this paper. (O)MP will denote cases referring to both these algorithms.

Let $D$ be a dictionary described in Section III-A containing a total of $Q$ tonic and phasic atoms from either the sigmoid-exponential, Bateman, or chi-square functions, i.e., $f \in \mathbb{D}_1$, or $f = D_2$, or $f = D_3$, and $g_{\alpha}$ an atom from the dictionary, i.e., $g_{\alpha} \in \mathbb{D}$: $c_\alpha \in \mathbb{B} \cup \Gamma^{(1)}$, or $c_\alpha \in \mathbb{B} \cup \Gamma^{(2)}$, or $c_\alpha \in \mathbb{B} \cup \Gamma^{(3)}$. According to MP [22], the initial signal $f$ can be written as a linear combination of infinite atoms as follows:

$$f = \sum_{n=0}^{\infty} c_n g_{\alpha_n}$$  \hspace{1cm} (2)

where $c_n$ are the atom coefficients. MP selects a set of atoms from $D$ that best matches the signal structure. If $g_{\alpha} \in \mathbb{D}$ is the atom at the first iteration, the signal $f$ can be written as follows:

$$f = (f^T g_{\alpha}) \cdot g_{\alpha} + Rf$$  \hspace{1cm} (3)

where $Rf$ is the signal residual after approximating $f$ in the direction $g_{\alpha}$. The selection of $g_{\alpha}$ is greedily performed in order to maximize the similarity of the atom to the original signal, i.e., such that $f^T g_{\alpha}$ is maximum. Note that in the selection criterion, we did not use the absolute value, as in the original MP algorithm. If we allowed negative inner product values, i.e.,
negative atom coefficients, the shape of the predefined atoms would change, resulting in a totally different signal interpretation. Negative coefficients of the SCR atoms, for example, would not have any physical meaning and would disturb the structure of the original EDA signal.

After the first iteration, residual $Rf$ is similarly decomposed by selecting an atom from dictionary $D$ that best matches its structure. Iteratively, the residual $R^{n+1}f$ can be written as follows:

$$R^{n+1}f = R^nf - ((R^nf)^T g_{c_n}) \cdot g_{c_n}$$

(4)

where $g_{c_n} \in D$ is selected to maximize $(R^nf)^T g_{c_n}$. After $N$ iterations, the original signal $f$ is approximated by $f_N$:

$$f_N = \sum_{n=0}^{N-1} \left[ \delta_n \cdot ((R^nf)^T g_{\delta_n}) \cdot g_{\delta_n} + (1 - \delta_n) \cdot ((R^nf)^T g_{\gamma_n}) \cdot g_{\gamma_n} \right]$$

(5)

where $\delta_n \in \{0, 1\}$ selects either a tonic or a phasic atom (see Table I), $\beta_n \in B$ and $\gamma_n \in \Gamma^{(1)}$, or $\gamma_n \in \Gamma^{(2)}$, or $\gamma_n \in \Gamma^{(3)}$ are the parameters of the $n$th atom.

Since processing of long duration signals is involved, we perform segmentation and frame-by-frame analysis. Let the superscript $(k)$ denote the $k$th time frame of length $L$, where $k = 1, \ldots, K$ with $K$ being the total number of frames. Therefore, the $k$th frame of the original signal in terms of the atoms selected by MP can be approximated by

$$f_N^{(k)} = \sum_{n=0}^{N-1} ((R^f)^{(k)}(k)g_{c_n}) \cdot g_{c_n},$$

(6)

We use a step of $\frac{L}{4}$ samples in order to avoid discontinuities introduced by the start and end of the signal frame. As will be described in Section III-C, SCR detection is based on the middle part of each signal frame for the same reason. A similar approach has been followed with other biosignals [39].

OMP [23], [31], [35], [40] is a refinement of MP which adds a least-squares minimization to obtain the best approximation over the atoms that have already been chosen at each iteration. Let $D_{SN} = [g_{c_0}, \ldots, g_{c_{N-1}}] \in \mathbb{R}^{L \times N}$ be the matrix containing the selected atoms after $N$ iterations. The least-squares approximation $f_N$ of signal $f$ is

$$f_N = D_{SN} (D_{SN}^T D_{SN})^{-1} D_{SN}^T f$$

(7)

where the term in brackets corresponds to the $N$ nonzero coefficients used to scale the selected atoms. OMP updates the residual by projecting the original signal onto the linear subspace spanned by the selected atoms; therefore, it never selects the same atom twice. Similar expressions hold for the signal of the $k$th time frame.

A schematic representation of the way OMP sparsely decomposes a signal frame is presented in Fig. 3. The original and reconstructed signals are shown with solid and dashed lines [see Fig. 3(a)]. OMP first selects a straight line atom in order to represent the signal level and then two phasic atoms centered around the 100th and 220th sample to capture the two SCRs. The original normalized atoms as well as the resulting phasic atoms when scaled with the coefficients computed by OMP are shown in Fig. 3(b) and (c). We also notice that the phasic atom centered around sample 100 has the highest energy [see Fig. 3(c)], since it represents the wider SCR of the frame. Finally, the third selected phasic atom is centered toward the end of the first annotated SCR around sample 140. This happens because the first phasic atom did not capture the entire energy of the corresponding SCR; therefore, another atom...
with much smaller coefficient was introduced. In Section III-C, we will describe how we combine these two phasic atoms located at samples 100 and 140 in order to reliably represent an SCR.

C. SCR Detection

Let \( \{g_{m}^{(k)}; m = 1, \ldots, M\} \) be the phasic atoms selected after \( N \) (O)MP iterations at the \( k \)th signal frame with \( M < N \), as at least one tonic atom will compensate for the signal level. Since the dictionary is predetermined, we know exactly the location of the maximum amplitudes of the phasic atoms, denoted as \( p_{m}^{(k)} \). As shown in Fig. 3 for atoms centered around samples 100 and 140, more than one phasic atom might be selected to represent a single SCR because the fit between dictionary atoms and signal is not always perfect. Taking advantage of their time proximity, we can group phasic atoms according to their location with a histogram of the \( p_{m}^{(k)} \) values with \( N_{h} \) bins. Each nonempty histogram bin \( h \) will contain all phasic atoms \( \{g_{m_{h}}^{(k)}\} \) capturing one SCR, which is represented by the linear combination of the grouped phasic atoms:

\[
\text{SCR}_{h}^{(k)}(t) = \sum_{\{g_{m_{h}}^{(k)}\}} c_{m_{h}}^{(k)} g_{m_{h}}^{(k)}(t). \tag{8}
\]

The weights \( c_{m_{h}}^{(k)} \) correspond to the atom coefficients computed by (O)MP. Because of the simple shape of the above SCR signal, we are able to find its amplitude and time of peak by simply locating its maximum point. SCRs with amplitude less than a predetermined threshold \( d_{\text{thr}} \) were omitted [8], in accordance to the manual annotation (see Section IV-B).

To estimate the final SCR locations for the whole signal, we merged the SCRs from all analysis frames. Since the starting and ending samples of each frame are more prone to errors due to discontinuity issues, we only take into account SCRs located in the middle \( \frac{L}{2} \) samples of each frame. The value \( \frac{L}{2} \) was chosen to coincide with the length of the analysis step in order to ensure that each SCR is taken into account once. For the first and last frame we also consider SCRs from the first \( \frac{L}{2} \) and last \( \frac{L}{2} \) samples, respectively. A signal frame of 10 s (320 samples), for example, will result in analysis step of approximately 3.3 s (107 samples), in which the shape of an SCR is easily identifiable, as shown in Fig. 3.

Since our approach involves local decisions made on each signal frame, a postprocessing step that takes into account the global EDA shape is necessary. Simple peak detection was performed and the located SCRs were then mapped to the nearest detected peak within a distance threshold \( d_{\text{thr}} \). SCRs not paired with a neighboring peak were omitted and those matched to the same peak were only counted once. In Fig. 3, for example, the phasic atoms centered around samples 100 and 140 were grouped together based on the proximity of their location and were further mapped to the peak of sample 110 to represent the first SCR. In order to capture the second SCR, the phasic atom centered around sample 220 was mapped to the peak from sample 230.

D. Description of the Least-Squares Fit Model for EDA

We compare our approach against the least-squares fit EDA representation model by Lim et al. [17], since it is conceptually the closest to our method involving a parameterization of the signal. The authors proposed a eight-parameter model that incorporates the signal tonic level, at most two SCRs and a recovery phase. SCL is represented by a straight line with offset \( c \). SCRs are captured by sigmoid-exponential functions with common rise time \( t_{r} \) and decay time \( t_{d} \). They are located at time offsets \( T_{os_{1}}, T_{os_{2}} \), and have amplitudes \( g_{1}, g_{2} \), respectively. Finally, the recovery phase is assumed to follow the exponential decay of SCRs with the same time constant decay \( t_{d} \) and an initial amplitude \( a_{0} \). Therefore, the eight-parameter model representing an EDA signal frame can be written as follows:

\[
f(t) = c + a_{0} e^{-\frac{t}{t_{d}}} + f_{s}(t; g_{1}, T_{os_{1}}) + f_{s}(t; g_{2}, T_{os_{2}}) \tag{9}
\]

\[
f_{s}(t; g, T_{os}) = \frac{g \cdot e^{-\frac{t-T_{os}}{t_{d}}}}{1 + (1\frac{t-T_{os}}{t_{d}})^{-2}}, \quad i = 1, 2. \tag{10}
\]

The nonlinear least-squares fit [41] was used to estimate the model parameters from the data. Time offset and amplitude of SCRs were initialized from peak detection. If less than two peaks were detected in a signal frame, only the corresponding parameters were estimated and the remaining were set to zero.

E. Evaluation

Evaluation of the proposed model is performed with respect to the quality of signal reconstruction, the ability to capture EDA-specific characteristics and the compression rate.

1) Signal Reconstruction: Signal reconstruction is evaluated with the root mean square (RMS) error, defined as follows:

\[
\text{RMSE} = \sqrt{\frac{1}{K} \sum_{k=1}^{K} \frac{1}{L} \sum_{l=1}^{L} \left( f^{(k)}(l) - f^{(k)}_{N}(l) \right)^{2}} \tag{11}
\]

where \( f^{(k)}(l) \) denotes the value of the \( l \)th sample at the \( k \)th signal frame with \( k = 1, \ldots, K \) and \( l = 1, \ldots, L \); similarly for the reconstructed signal \( f^{(k)}_{N}(l) \) with \( N \) (O)MP iterations. Low RMS error values reflect high-quality signal representation.

2) Information Retrieval: Evaluation with RMS error provides a limited view of the representation quality, since an overly complex model may result in perfect representation of the existing data, but might perform poorly on unseen patterns [42]. One technique to avoid overfitting with greedy algorithms, such as (O)MP, is early stopping, that might result in a better estimate of the original signal without introducing too many erroneous or noisy entries [43], [44]. Indeed, our results (see Section V-A2) suggest that 4-7 (O)MP iterations provide the most reliable EDA representation. An alternative way is the use of quantitative and qualitative criteria, depending on the application that assess the amount of meaningful information captured by the proposed models [45], [46]. In this study, we measure the ability of our approach to detect SCRs, an important EDA component.
SCR detection is assessed in two ways, each containing complementary accuracy metrics.

We first quantify the quality of SCR representation based on the amount of detected SCRs and their distance from the human annotated reference ones. Let $S_r = \{p_{r_j}\}$ and $S_d = \{p_{d_i}\}$ be the sets of locations of real and detected SCRs, respectively. The absolute relative difference between the number of real and detected SCRs captures the portion of hand annotated SCRs that is correctly located by our system:

$$\text{RD} = \frac{|S_r| - |S_d|}{|S_r|}$$

(12)

where $|\cdot|$ denotes the cardinality of a set. In order to see how many of the detected SCRs are correct, we compute the mean distance of each detected SCR from the closest real one

$$\text{MinDist} = \frac{1}{M_d} \sum_{i=1}^{M_d} (p_{d_i} - \min_{p_{r_j}} |p_{d_i} - p_{r_j}|).$$

(13)

We further calculate precision and recall measures. If $S_{rad} = \{p_{d_i} : \exists j \text{ s.t. } \min_{p_{r_j}} |p_{d_i} - p_{r_j}| < d_{max}\}$ are the estimated SCRs whose distance to their closest real SCR is less than a maximum threshold $d_{max}$, precision equals to the portion of detected SCRs that are close to a real one within the distance boundary $d_{max}$: $\text{Precision} = |S_{rad}|/|S_r|$. Recall captures the fraction of hand-annotated SCRs that are detected from our system within the same boundary: $\text{Recall} = |S_{rad}|/|S_r|$. F-score is the harmonic mean of precision and recall, providing a single measure from these complementary ones:

$$\text{Fscore} = 2 \cdot \frac{\text{Precision} \cdot \text{Recall}}{\text{Precision} + \text{Recall}}.$$  

(14)

These goodness of detection measures improve as the distance boundary $d_{max}$ increases, since evaluation becomes more forgiving to errors. Note that the distance boundary $d_{max}$ used for evaluation of SCR detection is different from the distance threshold $d_{hr}$ for mapping SCRs detected from the phasic atoms to the signal peaks (see Section III-C).

One can observe that precision is related to the MinDist metric quantifying how close an estimated SCR is to a real one. On the other hand, recall is closer to RD if that captures the algorithm’s ability to reliably detect the hand-annotated SCRs.

SCR detection results (see Section V-A2) are reported with a tenfold cross validation by randomly splitting the data from the 37 subjects. At each fold, the training set was used to find the optimal combination of $d_{hr}$ and $N$ (see Section III-C) from a pool of parameters $d_{hr} \in \{32, 64, \ldots, 320\}$ samples and $N \in \{2, 3, 4, 5\}$ bins, selected to maximize the Fscore of the corresponding data, since this yields a combined measure of the amount of missed and erroneously detected SCRs. The average value of each metric on the test set is reported based on the optimal parameters of each fold, as found from the training set. This reduces the risk of overfitting and is more indicative of the performance of our algorithm on novel data.

3) **Compression Rate:** Compression rate is evaluated as the number of bits used for representing the EDA signal over 1 s with low values indicating better compression ability.

**IV. DATA DESCRIPTION**

**A. Data Collection**

Our data contain 10-min recordings of 37 children watching a preferred television show in order to sustain attention and keep the subject stationary. Two silver-silver chloride electrodes were placed on the index and middle finger of the child’s nondominant hand. These were connected to the BIOPAC MP150 system that recorded EDA at a sampling rate of 32 Hz. Details about the experiment can be found in [47].

**B. Preprocessing and Annotation**

High-frequency noise artifacts were removed with a low-pass Blackman filter [48] of 32 samples (1 s).

Annotation of SCRs was performed by an expert using the visualization capabilities of BIOPAC’s AcqKnowledge software. SCR scoring involved visual inspection of the denoised EDA signal within a 30-s time frame and detection of signal “peaks” with the typical steep rise and slow decay. Each SCR amplitude was visually determined (see Fig. 1) and a minimum threshold of 0.05 $\mu$S was set for an SCR to be eligible for annotation [8]. In order to ensure reliable annotations, 25% of the data were double coded, yielding interannotator agreement of 96%.

**V. EXPERIMENTS**

We evaluate the performance of our method based on the collected and annotated data (see Section IV) and compare it against the least-squares fit approach (see Section V-A). Results concern the mean of each evaluation metric over all subjects. We further discuss how the analysis frame length and the SCR detection parameters affect our results (see Sections V-B and V-C).

In the following experiments, the maximum number of (OMP) iterations was determined so that the coherent signal structures, i.e., the nonzero signal components, are likely to be recovered. As explained in [22], this occurs when the residual decay ratio $r = 2 \cdot \log_{10}(\|\text{Residual}\|/\|f\|)$ decays at a constant rate. Our data suggest that this occurs after 15 OMP iterations with a dictionary of Bateman phasic atoms, while after
A. EDA Representation Results

For consistency with [17], in which 10-s isolated EDA segments were evaluated, results in this section are reported and compared to [17] with the same analysis frame.

1) Signal Reconstruction: During the first iteration, MP yields better signal reconstruction than OMP (see Fig. 5(a)). Since MP does not reestimate the atom coefficients, it selects more tonic atoms in order to adjust signal levels after the addition of new phasic ones. On the contrary, OMP readjusts the levels of the already selected tonic atoms. It is also apparent from Fig. 6 that tonic atoms are mostly selected by OMP during the first two iterations, whereas this occurs very often in MP. Therefore, during the first iteration, MP tends to capture part of the signal energy that is more attributed to the signal level and less to the signal fluctuations, resulting in lower RMS values. As more iterations occur, OMP achieves lower RMS indicating its ability to more reliably capture the signal variability.

2) Information Retrieval: Since OMP introduces more phasic atoms than MP for a given iteration, the absolute relative difference between real and estimated SCRs $RDiff$ is lower and becomes optimal around 4–7 iterations (see Fig. 5(b)). This can be also justified by the fact that in a 10 s frame, we have on average 1–3 tonic and 2–4 phasic atoms, the latter capturing the 1–3 expected SCRs. To achieve similar $RDiff$, MP needs more iterations due to the tonic atoms it keeps selecting.

30 iterations decay happens at an even steeper constant slope (see Fig. 4). Similar results are obtained from other types of dictionaries, omitted for the sake of simplicity. For this reason, we examine 1–15 (O)MP iterations and in fact, our experimental results indicate that even a smaller number can achieve reliable representation (see Section V-A).
Because MP selects less phasic atoms than OMP, it tends to capture the most distinct SCRs, resulting in smaller distances \( \text{MinDist} \) from the estimated to the real SCRs [see Fig. 5(c)] and higher precision [see Fig. 5(d)]. For the same reason MP shows poorer recall [see Fig. 5(e)], as it misses many SCRs. Compared to the least-squares fit approach, \( \text{Fscore} \) is higher for both MP and OMP [see Fig. 5(f)], indicating the ability of our system to reliably detect SCRs.

3) Compression Rate: In a 10 s segment, we compute the number of encoding bits for the raw signal, the least-squares fit and the proposed models. Long integers and single-precision binary floating-point numbers of 32 bits are assumed. Since the original signal was recorded with 32 Hz, a 10 s signal segment is represented with 320 samples (1280 Bytes); therefore, its compression rate \( \text{CR}_{\text{raw}} = 1024 \) bits/s.

Lim et al. [17] use a 8-parameter model to represent a 10 s signal. Assuming that the time offset parameters \( T_{os1}, T_{os2} \) are long integers and the remaining parameters are floating points, we need 32 Bytes, resulting in \( \text{CR}_b = 25.6 \) bits/s.

In our approach, given a known dictionary, each atom is represented by its indexed location in the dictionary and the corresponding coefficient. A 16 bit integer can encode the location of the 25 983 atoms used in our experiments. Assuming that each atom coefficient is a single-precision floating point, \( N \) selected atoms can be encoded by \( (16 \cdot N + 32 \cdot N) \) bits, which is equivalent to compression rate \( \text{CR}_p = 4.8N \) bits/s.

The proposed model achieves compression rate between 14.2 to 213 times upon the original signal depending on the number of (O)MP iterations (see Fig. 7). Compared to the least-squares approach, signal decomposition with 2–5 atoms also yields lower compression rate (see Fig. 7) and lower RMS error [see Fig. 5(a)].

B. Effect of Analysis Frame Length

The different time scales over which tonic and phasic EDA components coevolve, the first being much slower, create the need of analyzing multiple window lengths. Since the tonic component of the signal lies in the frequency range between 0–0.05 Hz [49], the upper limit corresponding to 20 s, we examined EDA representation metrics of variable frame lengths between 5–30 s. For the sake of simplicity, our experiments for this section are performed with OMP based on Bateman phasic atoms, but no significant differences occur from other combinations.

Representation results for the various analysis windows are greatly influenced by the number of (O)MP iterations. Signal reconstruction based on a few selected atoms is better for shorter than longer analysis frames, while this difference converges as the number of OMP iterations increases [see Fig. 8(a)]. Similar patterns occur with SCR detection \( \text{Fscore} \), although the short-term nature of the phasic part makes short analysis frames more compelling with respect to that [see Fig. 8(b) and (c)].

C. Effect of SCR Detection Parameters

We analyze SCR detection with respect to the distance threshold \( d_{thr} \) used for mapping the estimated SCRs to the nearest signal peaks and with respect to the number of histograms bins \( N_b \) for grouping the selected phasic atoms of each analysis frame. As given in Section V-B, we report results using OMP with Bateman atoms.

When changing \( d_{thr} \), the absolute relative difference \( \text{RDiff} \) between the number of real and detected SCRs remains almost unaltered [see Fig. 9(a)]. This is not the case for their minimum average distance \( \text{MinDist} \), which decreases as \( d_{thr} \) becomes larger [see Fig. 9(b)], suggesting that more information about signal peaks benefits SCR detection.

The number of histogram bins \( N_b \) strongly affects the absolute relative difference \( \text{RDiff} \) between real and estimated SCRs, since the latter directly depends on the number of groups in which the phasic atoms are combined. In contrast, metric \( \text{MinDist} \) is not largely affected by this parameter.
VI. DISCUSSION

Our method involves the design of EDA-specific dictionaries that take into account the tonic and phasic parts of the signal and the use of sparse decomposition techniques to represent the EDA. Compared to previous EDA models, our approach incorporates the idea of sparsity and is quantitatively evaluated directly on the signal as well as its human annotated characteristics. We examined three different types of phasic atoms of similar nature. These provided equivalent results in terms of signal reconstruction and information retrieval metrics (see Fig. 5); therefore, no definite conclusions can be drawn with respect to the most suitable dictionary for representing EDA among these variants. A larger amount of data from variable sources could potentially address this further in the future.

Sparsity is important for representing biomedical signals, especially for efficient storage and transmission in ambulatory (mobile device based) applications. Parameterization of EDA in light of compression was considered by Lim et al. [17], who used a eight-parameter sigmoid exponential model. This effort has several constraints, as it assumes the existence of at most two SCRs within 10 s and is tested on predefined signal segments. In real-world applications, however, the shape of the signal is more unpredictable and can be better captured by the variability of the proposed EDA-specific dictionaries. Sparsity has also been examined by the DCM approach [16], that computes the onset and amplitude of spontaneous fluctuations every 2 s, resulting in 10 parameters for every 10 s. In our sparse EDA model, each atom is represented by its location in the dictionary and the coefficient to which it is multiplied. This results in 2N parameters for a 10 s segment, where N is the number of (O)MP iterations. Taking into account that 5 (O)MP iterations give adequately reliable results (see Fig. 5), the two algorithms are fairly equivalent with respect to compression rate. Sparse representations can be further used to represent other psychophysiological signals with characteristic structure in time, such as electroencephalogram (EEG), electrocardiogram (ECG), and photoplethysmogram (PPG).

Our approach offers also advantages in terms of quantitative evaluation criteria. Most previous studies were implicitly validated with statistical analysis of the resulting parameters. Lim et al. [17] visualized an example residual error and reported residuals below 5% of the signal amplitude, while Bach et al. [19] plotted an original example and reconstructed signal with no further quantitative results. Bach et al. [16] reported the negative log-likelihood of the mean sum of squares of the residual, referred as Log Bayes Factor (LBF), with DCM reaching LBF values around $-3 \times 10^5$. In their nonnegative deconvolution approach, Benedek et al. [15] examined signal reconstruction with average RMS error of 0.019 $\mu$S. In our study, OMP running for 15 iterations achieves mean RMS error of 0.015, 0.019, and 0.017 $\mu$S for dictionaries containing sigmoid-exponential, Bate- man, and chi-square phasic atoms, respectively. Although 15 iterations are not indicative of the representation quality of our algorithm, a smaller number of iterations appears more suitable for the inherent low-dimensionality of EDA (see Fig. 5), they give the least compressed representation in our setup and are more comparable with [15]. Despite the fact that comparisons are performed on different data and in the context of slightly different approaches, these results indicate a promising framework for our method. Since dictionaries were empirically designed, our future work will examine dictionary learning techniques for data-specific dictionaries that can improve signal reconstruction measures.

The high correlation between SCRs and sympathetic nerve activity bursts and their extensive use in various psychophysical studies [8] suggest that reliable SCR detection mechanisms are necessary. It is therefore essential to validate automatic scoring systems with human-annotated ground truth, since the increasing use of longitudinal data renders manual annotation techniques laborious and time consuming. To the best of our knowledge, only Storm et al. [25] compared the number of automatically detected and manually annotated SCRs with linear regression analysis. For 0.05 $\mu$S minimum SCR amplitude, the magnitude of linear regression ranged between 0.75 and 0.98 depending on the model parameters and the data. Although it is useful to compare the total number of real and estimated SCRs, information about their relative location is also important. One of the novelties of this paper lies in the use of SCR detection criteria assessing both the sensitivity and specificity of automatic SCR scoring. These include two complementary pair measures: 1) the relative absolute difference between the number of real and detected SCRs, the average distance of detected SCRs to their closest real one and 2) the precision and recall of SCR detection, yielding the Fscore. Fscore is 83.43% using six OMP iterations and a tolerance threshold of 1.25 s between real and estimated SCRs [see Fig. 5(f)], This improves upon [17], which achieves 75.1% Fscore for the same conditions, and suggests that our approach appears promising for an automatic system.

Continuous advances in wearable technology suggest that physiological models should go beyond traditional signal reconstruction and pattern detection to signal interpretation in the context of internal human state tracking that could lead to individualized assessment and intervention [3], [7]. Previous efforts have provided meaningful interpretations of EDA models in terms empirical expectations (e.g., public versus non-public speaking [19], baseline versus anticipation [19], different noise stimuli [15], neutral, negative and positive arousing
images [16]). Other studies have analyzed EDA patterns in the context of human–computer interaction relative to individual’s affective states [6], [50]. The parameterized nature of the dictionary atoms in our approach can capture SCR characteristics, such as width, rise, and recovery time, which could further lead to more intuitive interpretations of EDA signals in terms of cognitive, social, and emotional stimuli. Data that allow these types of comparisons, including in the context of clinical translation such as therapy for Autism [51], will be explored as part of our future work.

A common problem when measuring changes in EDA is overlapping SCRs. It is typically agreed that when a second response occurs before completion of the first, one would count two overlapping responses [30]. Such an example is shown in Fig. 3(a) with two overlapping SCRs at samples 110 and 230, which we notice that our sparse decomposition approach is capable of discriminating. Previous studies have also assessed this by considering the underlying processes that contribute to SCRs as a linear time invariant system and measuring the event onsets of the system generator [27], [28].

VII. CONCLUSION

We propose a knowledge-driven EDA model that sparsely decomposes the signal into a small number of atoms from a dictionary. We focus on the dictionary design, which contains tonic and phasic atoms relevant to the signal levels and SCR fluctuations, respectively. With appropriate postprocessing, we automatically retrieve the SCRs from the EDA. Evaluation of our approach includes signal reconstruction, SCR detection, and compression rate analysis. These measures are improved upon the least-square fit model, that uses a parameterized template, providing benefits in terms of all criteria.

REFERENCES

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